COSMOLOGICAL IMPLICATIONS OF CLASSICAL KALUZA RELATIVITY

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ABSTRACT

The scalar field of classical Kaluza five-dimensional relativity, identified with the gravitational constant, should be interpreted in cosmological terms. The Kaluza equations with a Robertson-Walker metric imply that the gravitational constant is related to the cosmological electromagnetic energy density. For a homogeneous and isotropic universe, the gravitational constant, G, depends only on the Robertson-Walker time coordinate, t, and must scale as $G \propto t^2$. The Friedmann equation reduces to the classical four-dimensional form but the cosmological scale factor, $a(t)$, for a radiation-dominated universe scales as $a \propto t$. The gravitational constant is also the coupling constant between gravitational and electromagnetic fields in this classical unified theory, providing a universal charge-to-mass ratio and setting the magnitude of the fifth component of a covariant energy-momentum-charge five-vector.

Subject headings: cosmological parameters — cosmology: theory — gravitation — relativity

1. Introduction

A reconsideration of the implications of classical Kaluza relativity (Kaluza 1921) for the coupling of electrodynamics and gravity is warranted for a couple of reasons. One is of course that Kaluza relativity provides an elegant and compelling unification of those classical theories with an economy of assumptions. Another is that despite an intensive effort dating back to the advent of the quantum revolution, general relativity has defied unification with quantum theory. A third is that cosmology is in a golden age of discovery thanks to the sophisticated modern observing platforms on earth and in space. Unprecedented discoveries, such as the acceleration of the Hubble expansion, are being made which challenge our understanding of cosmology. Since classical Kaluza theory does make some interesting predictions on the cosmological scales, it is worth considering these results in the context of modern cosmological observations.

The essence of classical Kaluza (1921) relativity is to posit a five-dimensional (5D) metric, implying the existence of a fifth dimension besides the four dimensions of spacetime (4D). The electromagnetic vector potential provides the off-diagonal elements of the 5D metric. The new fifth diagonal element of the metric is a scalar field which is similar to the scalar field of Brans-Dicke relativity, inviting identification with the gravitational constant.

Applying such a 5D metric to the 5D Einstein equations yields both the 4D Einstein equations and the 4D vacuum Maxwell equations. Applying the 5D metric to a 5D geodesic hypothesis yields the usual 4D equation augmented with the Lorentz force law.

In fact, the Kaluza equation provide many more degrees of freedom than is necessary to reproduce the 4D field equations and 4D equations of motion. Therefore Kaluza made a practical assumption, the cylinder condition, that the 5D metric did not depend on the fifth coordinate. This simplifies the equations enormously, but there are still additional terms involving the scalar field that are not present in the standard 4D equations of motion. Even Brans-Dicke theory, which allows a contribution from the scalar field to the 4D field equations, assumes that no such contribution could exist in the 4D equations of motion without violating the equivalence principle.

So Kaluza eliminated the additional terms by assuming that the scalar field was constant. This was later realized to be problematic when the field equations are viewed self-consistently. In this paper it is shown that a constant scalar field is indeed a valid approximation on non-cosmological scales, arising merely from the lengthscale introduced by demanding correpondence with the 4D Einstein equations. Therefore, constancy of the scalar field necessary to recover the usual equations of motion need not be imposed as an *a priori* assumption, but is a natural implication of Kaluza relativity.

Since the scalar field is cosmological in its variation, Kaluza relativity is considered within the framework of the Robertson-Walker metric, assuming that the scalar field depends only on the Robertson-Walker time coordinate. The Kaluza version of the Friedmann equation is obtained and its implications discussed.

In the following development, the cylinder condition is retained as an *a priori* assumption. Starting with Klein (Klein 1926a,b), much of the subsequent work in what came to be known as Kaluza-Klein theory has attempted to ascribe the cylinder condition to compactification of the fifth dimension down to unobservable microscopic scales. This concept of compactification has been a feature of subsequent higher-dimensional Kaluza-type unification theories which focus on unification of quantum forces. Another approach to the cylinder condition has made use of projective theories, in which the fifth dimension appears as a mathematical artifice derived from the usual four dimensions.

In more recent work, Overduin & Wesson (1998) have revisited the compactified and projective approaches and compared them to Kaluza's original approach of treating the fifth dimension as a classical macroscopic dimension, which Overduin & Wesson call the non-compactified approach. While the compactified approach has been standard among workers in higher-dimensional relativity, Overduin & Wesson (1998) find no observational evidence to prefer a compatified or projective interpretation over a non-compactified one. They go on to explore the extra degrees of freedom arising from relaxation of the cylinder condition. The same non-compacified approach is taken here. But the justification for the cylinder condition is purely as the minimal form of the theory which reproduces standard 4D theory. The objective is to explore the cosmological implications of a 5D metric which is constant as function of the fifth dimension.

This paper enunciates the properties of the fifth component of the proper velocity and its theoretical context. One finds that that the gravitational constant provides a cosmic charge-to-mass ratio as well as dimensional characterization of the fifth dimensional coupling between electromagnetic and gravitational effects. Electric charge is the fifth-dimensional projection of an energy-momentum-charge five-vector and therefore not strictly conserved, implying the existence of a rest charge. The standard 4D equations of motion hold for any cosmological epoch, preserving the equivalence principle.

2. Classical Kaluza Equations

This section collects and summarizes the equations of classical Kaluza theory, with a macroscopic fifth dimension and assuming the cylinder condition, defined below.

The following notation is adopted. Five-dimensional (5D) tensors are indicated with a tilde to distinguish them from four-dimensional (4D) ones. The time coordinate is x^0, and the spatial coordinates x^1, x^2, x^3. The fifth coordinate is x^5. Summation is implied on repeated pairs of covariant and contravariant indices. Roman indices range over all five coordinates, and greek indices over the usual four coordinates of space and time. Partial derivatives $\partial/\partial x^a$ are abbreviated ∂_a.

Classical Kaluza relativity comprises 3 sets of equations: the form of the 5D metric, the 5D Einstein equations for the metric, and the 5D geodesic hypothesis for the metric. In a space described by the 5D metric $\widetilde{g}_{ab}$, particles of matter move along 5D paths described in terms of a constant b, and a 5D proper time $\widetilde{\tau}$:

$$\widetilde{g}_{ab}dx^a dx^b \equiv b^2 d\widetilde{\tau}^2, \tag{1}$$

which constrains the 5D proper velocity, $\widetilde{U}^a \equiv dx^a/d\widetilde{\tau}$. The signature of the fifth dimension in the metric must be spacelike (Bergmann 1942; Bargmann 1957). The entire space x^a is hyperbolic, but the subspace of x^1, x^2, x^3, x^5 is Euclidean.

The 4D metric $g_{\mu\nu}$ is related to the 4D proper time τ such that $g_{\mu\nu}dx^\mu dx^\nu = c^2 d\tau^2$, where c is the speed of light. The five-dimensional metric $\widetilde{g}_{ab}$ is given in terms of the 4D metric $g_{\mu\nu}$ and the electromagnetic vector potential A^μ:

$$\begin{aligned} \widetilde{g}_{\mu\nu} &= g_{\mu\nu} + k^2\phi^2 A_\mu A_\nu \qquad \widetilde{g}^{\mu\nu} = g^{\mu\nu} \\ \widetilde{g}_{5\nu} &= k\phi^2 A_\nu \qquad \widetilde{g}^{5\mu} = -kA^\mu \\ \widetilde{g}_{55} &\equiv \phi^2 \qquad \widetilde{g}^{55} = \frac{1}{\phi^2} + k^2 A^2 \end{aligned} \tag{2}$$

where $A^2 \equiv A_\alpha A^\alpha = g_{\alpha\beta}A^\alpha A^\beta$, $\widetilde{g}_{ab}\widetilde{g}^{bc} = \delta_a^c$, and k is a constant which will be seen to scale with the gravitational constant. The forms of (1) and (2) imply a relation between the 4D and 5D proper times:

$$b^2 = c^2 \left(\frac{d\tau}{d\widetilde{\tau}}\right)^2 + \phi^2 \left(kA_\nu \widetilde{U}^\nu + \widetilde{U}^5\right)^2. \tag{3}$$

The price of unifying $g_{\mu\nu}$ and A^μ within the framework of a 5D metric is the necessary introduction of the scalar field ϕ. The 5D metric (2) was studied in a series of classic papers including Kaluza (1921), Klein (1926a), and Thiry (1948). Closely related papers included Klein (1926b), Jordan (1947), and Brans & Dicke (1961). Bergmann (1942), Bargmann (1957), Applequist et al. (1987), and Overduin & Wesson (1998) provide reviews. Applequist et al. (1987) provide English translations of Kaluza (1921) and Thiry (1948). The Kaluza metric (2) reproduces the Lorentz force from the 5D equations of motion as well as the Maxwell equations from the 5D field equations. It is a marvel of Kaluza relativity that the same metric has the right behavior in the two independent sets of equations.

The simplest form of Kaluza theory, and the one originally put forth by Kaluza, assumes $\partial\widetilde{g}_{ab}/\partial x^5 = 0$, the relation historically known as the cylinder condition. This condition is assumed *a priori* here merely as the minimal version of Kaluza theory necessary to recover standard 4D physics. Overduin & Wesson (1998) explore the implications of relaxing the cylinder condition.

The 5D field equations are obtained from the 5D Einstein equations, $\widetilde{G}_{ab} \equiv \widetilde{R}_{ab} - \widetilde{g}_{ab}\widetilde{R}/2 = 0$, where $\widetilde{R}_{ab}$ is the 5D Ricci tensor, and $\widetilde{R}$ is the 5D scalar curvature. Kaluza originally assumed ϕ to be constant and it was not until some twenty five years later that Thiry (1948) obtained the full 5D field equations:

$$\widetilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{k^2\phi^2}{2} T^{EM}_{\mu\nu} - T^{\phi}_{\mu\nu}, \tag{4}$$

$$\nabla^{\mu} F_{\mu\nu} = -3g^{\mu\alpha} F_{\mu\nu} \partial_{\alpha} \ln \phi, \tag{5}$$

$$\phi^{-1} g^{\alpha\beta} \nabla_{\alpha} \nabla_{\beta} \phi = \frac{k^2\phi^2}{4} F_{\alpha\beta} F^{\alpha\beta}, \tag{6}$$

where ∇_{α} is the 4D covariant derivative, $G_{\mu\nu}$ is the standard 4D Einstein tensor, $F_{\mu\nu} \equiv \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, $T^{EM}_{\mu\nu}$ is the standard electromagnetic stress tensor, and $T^{\phi}_{\mu\nu} \equiv \phi^{-1}[\nabla_{\mu}\nabla_{\nu}\phi - g_{\mu\nu} g^{\alpha\beta} \nabla_{\alpha}\nabla_{\beta}\phi]$.

Independent of the field equations, the 5D equations of motion obtain from the 5D geodesic hypothesis:

$$\widetilde{U}^b \widetilde{\nabla}_b \widetilde{U}^a \equiv \frac{d\widetilde{U}^a}{d\widetilde{\tau}} + \widetilde{\Gamma}^a_{bc} \widetilde{U}^b \widetilde{U}^c = 0, \tag{7}$$

where $\widetilde{\nabla}_a$ is the 5D covariant derivative. $\widetilde{\Gamma}^a_{bc}$ is the 5D affine connection, which has the standard relation to the metric:

$$2\widetilde{\Gamma}^a_{bc} = \widetilde{g}^{ad} \left[\partial_c \widetilde{g}_{bd} + \partial_b \widetilde{g}_{cd} - \partial_d \widetilde{g}_{bc}\right]. \tag{8}$$

The modified 4D equations of motion are given by the 4D spacetime components of equation (7), rewritten in terms of the 4D proper velocity $U^{\nu} \equiv dx^{\nu}/d\tau$:

$$\frac{dU^{\nu}}{d\tau} + \widetilde{\Gamma}^{\nu}_{55}(U^5)^2 + 2\widetilde{\Gamma}^{\nu}_{5\mu} U^5 U^{\mu} + \widetilde{\Gamma}^{\nu}_{\alpha\beta} U^{\alpha} U^{\beta} + \left(\frac{d\widetilde{\tau}}{d\tau}\right)^2 U^{\nu} \frac{d^2\tau}{d\widetilde{\tau}^2} = 0. \tag{9}$$

Equation (9) is the generalized spacetime equation of motion.

3. Cosmological Significance of the Scalar Field

In Kaluza theory the constant k is fixed by correspondence of the 5D vacuum Einstein equations to the 4D Einstein equations. The spacetime components of equation (4) include source terms in $g_{\mu\nu}$ from both the electromagnetic field and ϕ. Correspondence is demanded between equation (4) and the usual 4D field equations with an electromagnetic source,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T^{EM}_{\mu\nu}, \tag{10}$$

so equations (4) and (10) imply

$$\phi^2 k^2 = \frac{16\pi G}{c^4}. \tag{11}$$

Fitting the scalar field into a coherent framework of understanding that still provides the 4D classical limits has long been a challenge to Kaluza relativity. In fact, as summarized by Applequist et al. (1987), the significance of ϕ in the field equations was not fully appreciated until some two decades after Kaluza's paper. Although Kaluza originally assumed ϕ constant, it was not realized until Thiry (1948) obtained the full field equations the implied constraint (6) on the electromagnetic field from assuming constant ϕ. Upon closer inspection, the magnitude of k given by equation (11) implies that setting equation (6) to zero is not a meaningful constraint on terrestrial electromagnetic fields and that constant ϕ is a valid assumption on non-cosmological scales.

The source term for ϕ on the right hand side of equation (6) is a Lorentz scalar representing the difference of the energy densities in the electric and magnetic fields. For electromagnetic fields in which energy is partitioned equally among electric and magnetic components, this term will vanish. But consider situations of strong astrophysical electromagnetic fields which may maximize this term. The field equations (6) for ϕ and the expression (11) for k imply a scale of variation for ϕ given by $l_\phi \sim (k^2\phi^2\epsilon_{EM})^{-1/2} \sim c^2/7(G\epsilon_{EM})^{1/2}$, where ϵ_{EM} is electromagnetic energy density. The scale of variation of ϕ arising from a neutron-star magnetic field of 10^{12} Gauss would be of order 1 astronomical unit. To achive variations in ϕ on a scale of kilometers would require energy densities of order 10^{38} erg cm^{-3} or magnetic field strengths of order 10^{20} G. For an electromagnetic energy density similar to the cosmic microwave background value of 0.25 eV cm^{-3}, the magnitude of k implies ϕ varies on a lengthscale similar to the radius of the universe.

So ϕ can be approximated as a constant in the equations of motion of terrestrial objects, and the constraint (6) from assuming constant ϕ is understood to be only on the cosmological electromagnetic energy density. Kaluza's original assumption of constant ϕ is indeed valid in the equations of motion of terrestrial objects and provides no constraint on non-cosmological electromagnetic fields.

The 4D Einstein equations (4) with the extra source term arising from ϕ, along with the field equation (6) for ϕ, are quite similar to the standard Brans-Dicke theory which augments general relativity with a scalar field. As suggested by Brans & Dicke (1961) and Jordan (1947), and implied by (11), ϕ may be interpreted in terms of the gravitational constant G. Specifically,

$$k^2 \equiv \frac{16\pi G_0}{c^4} \qquad \phi^2 = G/G_0, \tag{12}$$

where G_0 is a normalizing constant.

If ϕ, and therefore G, is truly a cosmological field, let us consider the field equations (4) and (6) when $g_{\mu\nu}$ is the standard Robertson-Walker metric, describing a homogeneous and isotropic universe. Further limit consideration to that of a flat universe, consistent with current observations. The non-zero components of $g_{\mu\nu}$ are:

$$g_{tt} = c^2 \qquad g_{rr} = -a^2 \qquad g_{\theta\theta} = -a^2 r^2 \qquad g_{\psi\psi} = -a^2 r^2 \sin^2\theta$$

where r, θ, and ψ are comoving spherical coordinates, t is the cosmological proper time, and $a(t)$ is the expansion scale factor. Demand ϕ is homogeneous and isotropic so that $\phi = \phi(t)$ only.

Then the $\widetilde{G}_{tt}$ component of equation (4) reproduces the Friedmann equation for a flat universe, but with a varying gravitational constant:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_0}{3c^2}\phi^2 \epsilon_{EM}, \tag{13}$$

where $T_{tt}^{EM} \equiv \epsilon_{EM}$. Under these assumptions for $\widetilde{g}_{ab}$, $T_{tt}^{\phi} = 0$ and makes no contribution to the Friedmann equation.

The corresponding equation for $\phi(t)$ is provided by (6):

$$\frac{\ddot{\phi}}{\phi^3} = \frac{4\pi G_0}{c^2} F^2 \tag{14}$$

where $F^2 \equiv F_{\alpha\beta}F^{\alpha\beta}$ is also a function of t.

Since we have not considered matter, these equations only apply to the early radiation-dominated universe. The dominant electromagnetic factor in these equations arises from the cosmic microwave background (CMB), and its energy density $\epsilon_{CMB} \propto a^{-4}$. For an isotropic radiation field like the CMB, energy is expected to be partitioned equally among magnetic and electric field oscillations so that $F_{CMB}^2 = 0$. In this case, $\phi \propto t$ and the gravitational constant $G(t) \propto t^2$. The modified Friedmann equation (13) then implies $a(t) \propto t$, whereas the standard Friedmann result for a radiation dominated universe is $a(t) \propto t^{1/2}$.

Equation (6) therefore implies the gravitational constant G is a function of the cosmological electromagnetic field, and G_0 refers to its value in the present epoch of cosmological electromagnetic energy density. The scale of variation $l_\phi \propto t^{-1}$, implying that G was constant for each previous cosmological epoch.

While mathematically similar to Brans-Dicke theory, classical Kaluza relativity contains significant differences. One is that Brans & Dicke anticipated the scalar field would find its origin in a cosmological matter field, whereas the Kaluza theory finds the origin of ϕ in the cosmological electromagnetic field. Another qualitative difference is that Brans & Dicke assumed the scalar field confined its effects to the field equations and did not enter in the equations of motion, in order to preserve the principle of the equivalence of gravitation and inertia. The Kaluza equations of motion (9) must in general admit dependencies on ϕ, so section 5 will address this issue.

4. G AND THE NATURE OF ELECTRIC CHARGE

This section summarizes the cosmological implications of Kaluza relativity for electric charge. In the 4D equations of motion (9), the term linear in U^μ must correspond to the Lorentz force. Under the approximation of constant (cosmological) ϕ, we indeed find from (2) and (8) that

$$2\widetilde{\Gamma}^{\nu}_{5\mu} = k\phi^2 g^{\nu\alpha} F_{\mu\alpha}. \tag{15}$$

Equation (9) therefore implies that in order to recover the standard expression for the Lorentz force, electric charge q must be identified with U^5 such that

$$U^5 = \frac{q}{k\phi^2 m_0 c} = c\,\frac{q/m_0}{4(\pi G_0)^{1/2}}\left(\frac{G_0}{G}\right), \tag{16}$$

where m_0 is rest mass of the object in 4D motion and $k\phi^2$ is given by (12).

The gravitational constant takes on new significance in light of (16). The Gaussian units of electric charge are $m^{1/2}l^{3/2}t^{-1}$, where m denotes a mass unit, l a length unit, and t a time unit. The units of G are $l^3m^{-1}t^{-2}$, so the gravitational constant provides a universal charge-to-mass ratio.

Whereas relativity tells us that mass/energy arises from "motion" in time and momentum arises from motion in space, Kaluza relativity tells us that electric charge arises from "motion" along the 5th coordinate. Just as particle energy $\propto U^0$ and spatial momentum $\propto \mathbf{U}$ are frame-dependent, so is electric charge $\propto U^5$. Thus an interesting physical picture emerges for the concept of electric charge. The electrical charge of an object at rest in the laboratory arises from its motion in the fifth dimension. Furthermore, because its charge is only one component of an energy-charge-momentum 5-vector, electric charge is not a Lorentz scalar. Let us examine this issue more closely.

The cylinder condition allows recourse to a simple equation of motion for U^5. The equations of motion (7) can be written in a simple and completely general form for the covariant 5-velocity $\widetilde{U}_a = \widetilde{g}_{ab}\widetilde{U}^b$. Because the metric commutes with the covariant derivative

$\widetilde{\nabla}_a$, (7) and the antisymmetry in (8) imply:

$$\frac{d\widetilde{U}_a}{d\widetilde{\tau}} = \frac{1}{2}\widetilde{U}^b\widetilde{U}^c\frac{\partial\widetilde{g}_{bc}}{\partial x^a}. \tag{17}$$

This form of the equations of motion is in terms of a normal partial derivative instead of a covariant derivative, and expresses the conservation properties of the metric. There is a conserved quantity associated with each invariant coordinate.

The cylinder approximation therefore implies a conserved quantity along 5D worldlines:

$$\widetilde{U}_5 = \phi k A_\nu \widetilde{U}^\nu + \phi\widetilde{U}^5 = \left(\frac{d\tau}{d\widetilde{\tau}}\right)\phi(kA_\nu U^\nu + U^5) = \text{constant}. \tag{18}$$

This expression for $\widetilde{U}_5$ can be combined with (3) to show that $d\tau/d\widetilde{\tau}$ is constant under the approximation of the cylinder condition. The constancy of $d\tau/d\widetilde{\tau}$ in turn implies that electric charge, identified with U^5, is not conserved. Instead, U^5 can be altered by 4D motion in an electromagnetic field as shown in (18).

Equation (18) can be rewritten in conventional terms:

$$\frac{16\pi G_0}{c^3}m_0 A_\nu U^\nu + q = \text{constant}. \tag{19}$$

Electric charge is a frame-dependent quantity, and its variation with motion in electromagnetic fields is given by (19). For a relativistic proton with proper velocity $\sim 100c$, moving in a 10^6 Gauss magnetic field characterized by a lengthscale of 10 meters, the variation in charge is $\sim 10^{-38}$. Therefore, while Kaluza relativity predicts electric charge is not a true Lorentz scalar, its variation with 4D motion is indistinguishable under laboratory conditions from a Lorentz scalar.

5. Terrestrial Equations of Motion

We are now in a position to consider the other terms in the 4D equations of motion (9) and address their implications for the equivalence principle. In the preceding section, the term linear in U^μ was identified with the Lorentz term as long as U^5 was identified with electric charge. The values of U^5 for elementary particles turn out to be astronomical: $10^{18}c$ for protons, and $10^{21}c$ for electrons.

This creates an apparent problem in the equations of motion (9), one which was recognized by Kaluza. It is that the magnitude of U^5 for elementary charged particles would cause the term quadratic in U^5 to dominate the Lorentz and Einstein force terms linear and quadratic, respectively, in U^μ. However, under the assumption of constant ϕ, again implying epochal constancy of the cosmological electromagnetic energy density and of the gravitational constant G, the term in (9) quadratic in U^5 vanishes because $\widetilde{\Gamma}^\nu_{55} = 0$.

Under the approximation of constant ϕ for non-cosmological scales, the coefficient of the term quadratic in U^μ is given by:

$$\widetilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} + \frac{k^2\phi^2}{2} g^{\mu\nu} \left(A_\alpha F_{\beta\nu} + A_\beta F_{\alpha\nu} \right). \tag{20}$$

The first term on the right hand side of (20) is just the standard 4D affine connection. The second term in A^μ is the contribution of electromagnetic stresses to spacetime curvature and is of order k^2A^2.

The units of vector potential A are $m^{1/2}l^{1/2}t^{-1}$. Therefore $k^2A^2 \sim GA^2/c^4$ is the dimensionless number that expresses the strength of the coupling corrections to the 4-D theory. k^2A^2 is a very small number, and quantifies weak couplings to electromagnetic fields. For a neutron star magnetic field of strength 10^{12} G and lengthscale 10 km, k^2A^2 is of order 10^{-13}. For galactic electromagnetic field energy densities of 1 eV cm^{-3} and length scale 1 parsec, k^2A^2 is of order 10^{-32}.

The final term in (9) was shown to be zero in the previous section, implied by equations (18) and (3). With this, the 4D equations of motion (9) reduce to the standard forms when the scalar field is understood to depend only on the cosmological proper time. No terrestrial or astrophysical electromagnetic field can produce an observable deviation in the scalar field. Therefore the principle of the equivalence of gravitation and inertia is preserved by the Kaluza equations of motion on non-cosmological scales. As in standard Brans-Dicke theory, the scalar field appears only in the field equations and not in the equations of motion.

6. Conclusions

The elegant and compelling unification of electrodynamics and general relativity provided by classical Kaluza 5D relativity provides many more degrees of freedom than are necessary to recover standard 4D theory. Historically, the equations were simplified by assuming that the 5D metric does not depend on the fifth coordinate (the cylinder condition) and that the scalar field is constant. The latter condition was later seen to impose an unmotivated constraint on the electromagnetic field. While the cylinder condition is retained here as a simplifying assumption, the constancy of the scalar field actually follows from the lengthscales associated with identifying the scalar field with the gravitational constant. Specifically, the scalar field has no variation except perhaps on cosmological scales, and its constancy therefore imposes no constraint on any terrestrial or astrophysical electromagnetic fields.

A treatment of the 5D theory with a Robertson-Walker metric for the 4D sub space implies that, for a radiation-dominated universe, the Robertson-Walker scale factor is proportional to the Robertson-Walker time coordinate, t, and that the gravitational constant scales as t^2. Otherwise, the gravitational constant is truly constant for each cosmological epoch and provides the coupling constant between gravitational and electromagnetic fields. It also provides a universal charge-to-mass ratio and sets the magnitude of the fifth component of a covariant 5D energy-momentum-charge five-vector.

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