Electromagnetic antigravity

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Abstract

Discovery of a coupling between gravity and electromagnetism could unleash a phase of technological progress culminating in practical interstellar travel and the mastery of gravitational force. The classical Kaluza [Sitz. Preuss. Akad. Wiss. 966, (1921)] theory of five-dimensional relativity offers a compelling and elegant theoretical basis for such a coupling. The purely classical five-dimensional theory is revisited here without assumption of a microscopic or "rolled-up" fifth dimension, and is examined entirely from the standpoint of the equations of motion. Some of the key implications of the five-dimensional geodesic hypothesis are revisited and made explicit: that electric charge arises from motion in the fifth dimension, that the energy-momentum four-vector is seen to be the four-dimensional projection of an energy-momentum-charge five-vector, that electric charge is frame-dependent, and that the gravitational constant constitutes a cosmic charge-to-mass ratio. In new results, an interpretation is described of the scalar field characteristic of the five-dimensional theory which allows recovery of the four-dimensional equations of motion without unphysical constraints on the electromagnetic field. The Newtonian limit of the equations of motion are shown to contain electromagnetic effects which do not couple to electric charge, and these effects are quantified. Moreover, geodesics exist for charged bodies moving in electromagnetic fields which describe motion through space at constant time.

I. INTRODUCTION

Great technological revolutions followed each of the great revolutions in physical law: Newtonian mechanics, electrodynamics, relativity, and quantum theory. Each of those technological revolutions increased human mastery of the forces of nature. Yet, despite the technological revolutions of the past 100 years, one technological dream still lies unfulfilled: practical interstellar travel. While general relativity provides exquisite understanding of the structure and evolution of the universe, it pushes the dream of practical interstellar travel under the umbrella of exotic phenomena, such as worm holes and naked singularities. Enormous distance scales combine with the finite speed of light to confound practical interstellar travel and communication.

Four fundamental forces are at work in our universe and two of them, the strong and weak forces, manifest only at the scale of the atom. The other two, gravity and electromagnetism, are of infinite range, and accessible to human experience. While we are able to harness the force of gravity, it is only to a limited extent; hydroelectric power from captured gravitational potential energy, for example. But we do not create or manipulate gravitational fields. Rather, those fields comprise a spacetime background created by celestial bodies beyond human control.

It is only through the electromagnetic field that we really harness the forces of nature, for we can create and manipulate electromagnetic fields. Indeed, all of human life and technology exists on the electromagnetic plane: chemistry, manufacturing, power generation, communications. While we are able to harness atomic phenomena, through transistors or nuclear reactors, we still access these forces in manufacturing processes that ultimately arise from our mastery of the electromagnetic field. Therefore, if some aspect of physical law was ever discovered which would allow human mastery of gravity, and the accomplishment of practical interstellar travel, perhaps it would lie in a linkage between electromagnetism and gravity. In the context of this paper, electromagnetic antigravity refers to such control over gravity exercised through electromagnetic means.

Of course, explaining unity among the fundamental forces is an ongoing endeavor. A quantum theory of electrodynamics has been found, and it has been unified with the weak force with spectacular success in the Standard Model. There is every reason to believe the strong force can be theoretically unified with the weak and electromagnetic forces. In counterpoint to our success in unifying three of the four forces at the quantum level, there is one outstanding failure: a quantum theory of general relativity. Without this quantization, general relativity cannot be unified with the other 3 forces. Perhaps the breakthrough is imminent, but 75 years of effort have so far been fruitless. It is inescapable that physical law today is a house divided: general relativity and quantum theory. Only electrodynamics has a foot in both sides. Given the bicameral state of physical law, a classical unification of general relativity and electrodynamics may be of practical technological interest.

Kaluza [6] suggested a classical unification of general relativity and electrodynamics, which Einstein received only two years after his publication of general relativity. Kaluza's insight was that by extending the metric tensor to 5 dimensions, the five-dimensional (5D) equations of general relativity then yield both the equations of four-dimensional (4D) general relativity, and the equations of electrodynamics. The 4D spacetime metric is framed by the electromagnetic vector potential to comprise the 5D metric. The field equations and the equations of motion are two independent parts of general relativity, so it was stunning that the same assumed 5D metric, which yielded the Maxwell equations from the 5D Einstein equations, also yielded the Lorentz force law from the 5D geodesic hypothesis. The idea of writing physical law in higher dimensions has been a key concept underlying unified field theories to this day.

Kaluza's theory was published during the climax of the development of modern quantum theory by Schroedinger and Heisenberg. Classical electrodynamics could not explain the structure of the atom, and so it followed that unification of classical electrodynamics and general relativity, while mathematically interesting, could not be a valid path to greater understanding of both theories. Without quantum theory in the mix, the unified theory would be incomplete, if not altogether wrong. At least, that was the prejudice of the times.

In fact, this theory is typically known as Kaluza-Klein theory after the contribution by Klein [7,8]. Although Kaluza's original treatment was purely classical, Klein linked the 5D theory to the emerging quantum theories. Starting with Klein's paper, and up through the multi-dimensional generalizations of the last few decades [1], the extra dimensions have been treated as microscopic. Klein used the Kaluza theory to link electric charge to a "rolled up" closed fifth dimension. From a purely classical standpoint, however, there is no need to make this assumption. Because the citizens of Flatland cannot perceive a third spatial dimension, for example, does not mean that the third dimension is microscopic.

There is a kernel of truth to general relativity that confounds assimilation into quantum theory, and knowing that the key to mastery of gravity should involve a coupling to electromagnetism, it behooves us to look again at potential classical couplings between the two, and their implications for interstellar travel. That is the objective of this paper.

In the subsequent sections, the implications of the 5D geodesic hypothesis, the general relativistic equations of motion, are examined. The Kaluza metric is recovered from the equations of motion. The key results arising from the 5D geodesic hypothesis are enunciated: that electric charge arises from motion in the fifth dimension, that the energy-momentum four-vector is seen to be the four-dimensional projection of an energy-momentum-charge five-vector, electric charge is frame-dependent, and that the gravitational constant provides a cosmic charge-to-mass ratio. While these results are implicit in Kaluza's original paper, discussion of the equations of motion in the literature is limited compared to discussion of the field equations. Departing from the literature in this analysis, no assumption is made about the fifth dimension being microscopic. Indeed, the usefulness of a fifth dimension for interstellar travel relies on it being macroscopic. Rather, it is merely assumed no field variable depends on the fifth dimension, an assumption originally made by Kaluza. The lack of human perception of the fifth dimension is therefore due merely to all fields being constant along that coordinate.

Significant discussion is given to interpretation of the scalar field which is inescapable in Kaluza theory. Kaluza himself ignored variation of this field, but in subsequent decades it was shown that this field must be addressed in a coherent theory [1]. Here, an interpretation is suggested for this field which allows recovery of the 4D equations of motion without unphysical constraints on the electromagnetic field.

With an understanding of the role of the scalar field in the equations of motion, the key results of this paper are developed. One is the emergence of electromagnetic corrections to Newtonian gravity which do not couple to charge. Another is the identification of geodesics describing motion in space at constant time. Both of these effects, if practicable, would have implications for the human mastery of gravity, and the attainment of practical interstellar travel.

II. GEODESIC HYPOTHESIS IN FIVE DIMENSIONS

The following notation is adopted. Five-dimensional (5D) tensors are indicated with a tilde to distinguish them from four-dimensional (4D) ones. The time coordinate is x^0, and the spatial coordinates x^1, x^2, x^3. The fifth coordinate is x^5. Summation is implied on repeated pairs of covariant and contravariant indices. Roman indices range over all five coordinates, and greek indices over the usual four coordinates of space and time. Partial derivatives $\partial/\partial x^a$ are abbreviated ∂_a.

In a space described by the 5D metric $\widetilde{g}_{ab}$, particles of matter move along 5D paths described in terms of a constant b, and a 5D proper time $\widetilde{\tau}$:

$$\widetilde{g}_{ab} dx^a dx^b \equiv b^2 d\widetilde{\tau}^2 \tag{1}$$

which constrains the 5D length of the 5D proper velocity. Previous results [2,3] show that the signature of the fifth dimension in the metric must be spacelike. The entire space x^a is hyperbolic, but the subspace of x^1, x^2, x^3, x^5 is Euclidean.

The 5D equations of motion obtain from the 5D geodesic hypothesis:

$$\frac{d\widetilde{U}^a}{d\widetilde{\tau}} + \widetilde{\Gamma}^a_{bc} \widetilde{U}^b \widetilde{U}^c = 0 \tag{2}$$

where $\widetilde{U}^a \equiv dx^a/d\widetilde{\tau}$ and $\widetilde{\Gamma}^a_{bc}$ is the 5D affine connection, which has the standard relation to the metric:

$$2\widetilde{\Gamma}^a_{bc} = \widetilde{g}^{ad}\left[\partial_c \widetilde{g}_{bd} + \partial_b \widetilde{g}_{cd} - \partial_d \widetilde{g}_{bc}\right]. \tag{3}$$

The modified 4D equations of motion are given by the 4D spacetime components of (2):

$$\frac{d\widetilde{U}^\nu}{d\widetilde{\tau}} + \widetilde{\Gamma}^\nu_{55}(\widetilde{U}^5)^2 + 2\widetilde{\Gamma}^\nu_{5\mu}\widetilde{U}^5\widetilde{U}^\mu + \widetilde{\Gamma}^\nu_{\alpha\beta}\widetilde{U}^\alpha\widetilde{U}^\beta = 0. \tag{4}$$

Expressed in terms of the 4D proper time τ defined by $g_{\mu\nu}dx^\mu dx^\nu = c^2 d\tau^2$, where $g_{\mu\nu}$ is the 4D metric and c is the speed of light, (4) can be rewritten:

$$\left(\frac{d\tau}{d\widetilde{\tau}}\right)^2 \left(\frac{dU^\nu}{d\tau} + \widetilde{\Gamma}^\nu_{55}(U^5)^2 + 2\widetilde{\Gamma}^\nu_{5\mu}U^5 U^\mu + \widetilde{\Gamma}^\nu_{\alpha\beta}U^\alpha U^\beta\right) + U^\nu \frac{d^2\tau}{d\widetilde{\tau}^2} = 0. \tag{5}$$

Equation (5) is the generalized spacetime equation of motion.

III. THE KALUZA METRIC AND THE INTERPRETATION OF CHARGE

The Kaluza metric $\widetilde{g}_{ab}$ is completely determined by demanding $\widetilde{\Gamma}^\nu_{5\mu}$ take a form such that the term linear in U^ν corresponds to the Lorentz force vector (in Gaussian units),

$$\frac{q}{mc}F_{\alpha\beta}U^\beta g^{\alpha\nu}, \tag{6}$$

where $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$, A^α is the electromagnetic vector potential, m is particle mass, and q is particle charge. Equations (2), (4) and (5) show that electromagnetic forces enter the equations of motion linear in U^ν, while gravitational forces enter it quadratically. Electromagnetic antigravity refers to electromagnetic modifications of the gravitational term, quadratic in U^ν.

The five-dimensional metric $\widetilde{g}_{ab}$ is therefore given in terms of $g_{\mu\nu}$ and A^μ:

$$\begin{aligned}
\widetilde{g}_{\mu\nu} &= g_{\mu\nu} + k^2 A_\mu A_\nu/\phi \\
\widetilde{g}_{5\nu} &= kA_\nu \\
\widetilde{g}_{55} &\equiv \phi \\
\widetilde{g}^{\mu\nu} &= g^{\mu\nu}
\end{aligned} \tag{7}$$

$$\tilde{g}^{5\mu} = -kA^\mu/\phi$$
$$\tilde{g}^{55} = \frac{1}{\phi} + \frac{k^2A^2}{\phi^2}$$

where $A^2 \equiv A_\alpha A^\alpha = g_{\alpha\beta}A^\alpha A^\beta$, $\tilde{g}_{ab}\tilde{g}^{bc} = \delta_a^c$, and k is a constant. The price of unifying $g_{\mu\nu}$ and A^μ within this framework is the necessary introduction of the scalar field ϕ.

(7) is essentially the 5D metric studied in a series of classic papers by Kaluza [6], Klein [7], and Thiry [9]. There were also closely related papers by Klein [8], Jordan [5], and Brans & Dicke [4]. Bergmann [3] and Bargmann [2] provide reviews. Applequist, Chodos, and Freund [1] provide a historical summary, translations of [6] and [7] , and further references.

In addition to assuming the form of $\tilde{g}_{ab}$ to recover the Lorentz force, it is also necessary to identify the proper velocity in the fifth dimension with electric charge:

$$U^5 \equiv \frac{dx^5}{d\tau} = \frac{q/mc}{k}. \tag{8}$$

IV. THE FIELD EQUATIONS FIX k

Before obtaining modifications to the 4D equations of motion, the newly introduced quantities of k and ϕ must be addressed. The constant k is fixed by correspondence of the 5D vacuum Einstein equations to the 4D Einstein equations. The 5D Einstein equations,

$$\tilde{G}_{ab} \equiv \tilde{R}_{ab} - \frac{1}{2}\tilde{g}_{ab}\tilde{R} = 0, \tag{9}$$

where $\tilde{R}_{ab}$ is the 5D Ricci tensor, and $\tilde{R}$ is the 5D scalar curvature, are decomposed into the familiar 4D components [9]:

$$\tilde{G}_{\mu\nu} = G_{\mu\nu} - 2\pi k^2\phi T^{EM}_{\mu\nu} - T^{\phi}_{\mu\nu}, \tag{10}$$

$$\nabla^\mu F_{\mu\nu} = -3\phi^{-1/2}\partial^\mu\phi^{1/2}F_{\mu\nu}, \tag{11}$$

$$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\phi^{1/2} = \frac{k^2\phi^{3/2}}{4}F_{\alpha\beta}F^{\alpha\beta}, \tag{12}$$

where ∇_α is the covariant 4D derivative, $T^{EM}_{\mu\nu}$ is the electromagnetic stress tensor, and $T^{\phi}_{\mu\nu}$ is a collection of terms that depend on ϕ.

The spacetime components of (10) therefore include source terms in $g_{\mu\nu}$ from both the electromagnetic field and ϕ (10). Furthermore, k is determined by demanding correspondence between (10) and the 4D field equations with an electromagnetic source:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T^{EM}_{\mu\nu}, \tag{13}$$

so (10) and (13) imply

$$k^2 = \frac{4G}{c^4}. \tag{14}$$

The units of G are $l^3 m^{-1} t^{-2}$, while those of the vector potential A are $m^{1/2} l^{1/2} t^{-1}$, where m is used here to denote a mass unit, l a length unit, and t a time unit. Therefore GA^2/c^4 is the dimensionless number that expresses the strength of the coupling corrections to the 4-D theory.

For a neutron star magnetic field of strength 10^{12} G and lengthscale 10 km, GA^2/c^4 is of order 10^{-13}. For galactic field strengths of 1 eV cm^{-3} and lengthscale 1 parsec, GA^2/c^4 is of order 10^{-32}. The metric terms linear in kA are strong enough to produce the Lorentz force because U^5 is inversely proportional to k.

V. ENERGY-MOMENTUM-CHARGE 5-VECTOR AND G

Equations (8) and (14) imply the x^5 component of the proper velocity is

$$U^5 = c\,\frac{q/m}{2G^{1/2}} \equiv \alpha \frac{dh}{d\tilde{\tau}}. \tag{15}$$

The origin of electric charge finds its explanation in this theory as arising from motion in the 5th dimension. As particle energy and momentum are frame-dependent, so is electric charge. Just as the time coordinate $x^0 \equiv ct$ is normalized to units of length with the constant c, so is x^5 normalized with a constant $\alpha = c/2mG^{1/2}$. Therefore the fifth coordinate h has units of charge-time.

The gravitational constant takes on new significance in light of (15). The Gaussian units of electric charge are $m^{1/2} l^{3/2} t^{-1}$, so $G^{1/2}$ sets a universal charge-to-mass ratio. The values of U^5 for elementary particles are astronomical: $10^{18}c$ for protons, and $10^{21}c$ for electrons. This immediately creates an apparent problem in the equations of motion (5) because the term quadratic in U^5 would dominate the Lorentz and Einstein force terms. This term will find its reconciliation in the interpretation of ϕ, and preserve the 4D limit for elementary particles.

The constant b is obtained in the 5D Minkowski limit, when $\widetilde{g}_{ab}$ is diagonal and constant. The 5D equivalence principle demands that we be able to tranform to a coordinate system characterized by the 5D Minkowski metric, where x^5 shares the sign of the spatial dimensions. In the absence of electromagnetic or gravitational fields, and in the absence of spatial motion

$$\widetilde{g}_{ab}\widetilde{U}^a\widetilde{U}^b = b^2 = c^2\left[1 - \frac{(q_0/m)^2}{4G}\right]. \tag{16}$$

Since (15) implies electric charge is a frame-dependent quantity, (16) has specified a rest charge q_0 which is the electric charge of a particle at rest.

VI. CYLINDER CONDITION AND VARIATION OF CHARGE

The 5D metric (7) is rich enough in structure that general relativity and electrodynamics, along with significant coupling terms, can be recovered in the absence of any variation of $\widetilde{g}_{ab}$ with x^5. The simplest form of Kaluza theory therefore assumes $\partial\widetilde{g}_{ab}/\partial x^5 = 0$, a relation historically known as the cylinder condition. The cylinder condition is assumed here, and it allows recourse to a simple equation of motion for $\widetilde{U}^5$.

The equations of motion (2) can be written in a simple and completely general form for the covariant component of the 5-velocity $\widetilde{U}_a = \widetilde{g}_{ab}\widetilde{U}^b$. Because the metric commutes with the covariant derivative $\widetilde{\nabla}_a$, (2) and the antisymmetry in (3) implies:

$$\frac{d\widetilde{U}_a}{d\widetilde{\tau}} = \frac{1}{2}\widetilde{U}^b\widetilde{U}^c\frac{\partial\widetilde{g}_{bc}}{\partial x^a}. \tag{17}$$

This form of the equations of motion is in terms of a normal partial derivative instead of a covariant derivative, and expresses the conservation properties of the metric. There is a conserved quantity associated with each invariant coordinate.

The cylinder condition therefore implies a conserved quantity along 5D worldlines:

$$\widetilde{U}_5 = kA_\nu \widetilde{U}^\nu + \phi \widetilde{U}^5 = \left(\frac{d\tau}{d\widetilde{\tau}}\right)(kA_\nu U^\nu + \phi U^5); \qquad \frac{d\widetilde{U}_5}{d\widetilde{\tau}} = 0. \tag{18}$$

We assume for the moment that $d\tau/d\widetilde{\tau}$ is constant, and return to verify the assumption. Note that it is U^5 which is identified with electric charge in the equations of motion (5), while (18) shows that U^5 alone is not conserved. Charge evidently is affected by motion in electromagnetic fields, and there will be a rest charge. Equation (18) can be rewritten in conventional terms:

$$\frac{G}{c^3} m A_\nu U^\nu + q = \text{constant}. \tag{19}$$

Electric charge is a frame-dependent quantity, and its variation with motion is given by (19). For a relativistic proton with proper velocity $\sim 100c$, moving in a 10^6 Gauss magnetic field characterized by a lengthscale of 10 meters, the variation in charge is $\sim 10^{-30}$. Therefore, for elementary charged particles, the "rest" charge and its variation with motion are approximately equal and constant, and cancel out of the energy-momentum-charge relations (16), preserving the standard 4D energy-momentum relations.

VII. INTERPRETATION OF THE SCALAR FIELD

The mathematical cost of unifying general relativity and electrodynamics with a 5D metric is the introduction of the scalar field ϕ. In fact, as summarized by Applequist et al.[1], the significance of ϕ in the field equations was not fully appreciated until some two decades after Kaluza's paper. In order to calculate the 5D connections and the corresponding 5D equations of motion, the scalar field must be assimilated into a coherent framework of understanding that still provides the 4D classical limits.

The field equations for ϕ (12) and the expression for k (14) imply a scale of variation for ϕ given by $l_k \sim (k^2 e_{EM})^{-1/2} \sim c^2/(G e_{EM})^{1/2}$, where e_{EM} is electromagnetic energy density. For an electromagnetic energy density similar to the observed average galactic value of 1 eV cm^{-3}, the magnitude of k implies ϕ varies on a lengthscale similar to the radius of the universe. The scale of variation of ϕ arising from a neutron-star magnetic field of 10^{12} Gauss would be of order 1 astrononmical unit. To achive variations in ϕ on a scale of kilometers would require energy densities of order 10^{40} erg cm^{-3} or magnetic field strengths of order 10^{20} G.

Such variation is of interest only on cosmological scales, and not of interest for the equations of motion of terrestrial objects. Therefore ϕ can be approximated as a constant in the equations of motion, and the constraint on (12) from assuming constant ϕ is understood to be only on the cosmological electromagnetic energy density. As for Bran & Dicke [4] and Jordan [5], ϕ may be interpreted as the gravitational constant G, but it is not necessary for simplifying the equations of motion. Explaining cosmological variation in the gravitational constant as arising from variation in the cosmological electromagnetic energy density is, however, an intriguing idea.

The constancy of ϕ on terrestrial scales makes the identity exact between the 5D connection $\widetilde{\Gamma}^{\nu}_{5\mu}$ in (5) and the Lorentz force. Although Kaluza originally assumed ϕ constant, it was not realized until much later the implied constraint on the electromagnetic field. The simplification of constant ϕ, as for the cylinder condition, greatly simplifies the 5D connections, and therefore the equations of motion. Yet even under these constraints, couplings between gravity and electrodynamics emerge.

The constant of the motion $\widetilde{U}_5$ (18) allows a simple relation between the 5D and 4D proper times:

$$\left(\frac{d\tau}{d\widetilde{\tau}}\right)^2 = \frac{b^2}{c^2} - \frac{\widetilde{U}_5^2}{\phi c^2} \Longrightarrow \text{constant.} \tag{20}$$

This validates the earlier assumption that $d\tau/d\widetilde{\tau}$ is constant and also eliminates the final term in (5). $d\tau/d\widetilde{\tau}$ differs from 1 only for charged particles, or uncharged particles in electromagnetic fields.

VIII. REDUCED EQUATIONS OF MOTION

Return now to the 4D equations of motion (5) obtained from the 5D geodesic hypothesis. Under the assumptions of no spacetime variation in ϕ, and the invariance of $\widetilde{g}_{ab}$ with respect to x^5, the relevant 5D connections (3) reduce to:

$$\widetilde{\Gamma}^{\mu}_{55} = \widetilde{\Gamma}^{5}_{55} = 0 = \widetilde{\Gamma}^{a}_{55} \tag{21}$$

$$\widetilde{\Gamma}^{\mu}_{5\alpha} = \frac{k}{2} g^{\mu\nu} F_{\alpha\nu} \tag{22}$$

$$\widetilde{\Gamma}^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta} + \frac{k^2}{2} g^{\mu\nu}(A_\alpha F_{\beta\nu} + A_\beta F_{\alpha\nu}) \tag{23}$$

The term in (5) quadratic in U^5 therefore vanishes with its coefficient. The term linear in U^5 reduces exactly to the Lorentz force law. The remaining term modifies the traditional 4D geodesic hypothesis to include a contribution from electromagnetic fields. This electromagnetic contribution does not couple to electric charge, and its manifestation in the Newtonian limit is described next.

IX. NEWTONIAN LIMIT

The Newtonian limit consists of assuming the spatial components of the 4D proper velocity are much smaller than the time component, that the metric is approximately Minkowskian, and that there is no time dependence. Consideration is further restricted to uncharged particles, $q_0 = 0$.

The electromagnetic corrections make no contribution to $dU^0/d\tau$, and U^0 remains constant in the absence of any time dependence. The equations of motion (5) and (23) then reduce to the spatial dimensions:

$$\frac{d^2\mathbf{x}}{dt^2} + \nabla\left[\psi_g + \frac{2G}{c^2}\psi_e^2\right] = 0 \tag{24}$$

where ψ_g is the Newtonian gravitational potential, ψ_e is the electric potential, and $\mathbf{x}$ and ∇ are standard 3D spatial vectors.

It would take a steady voltage of order 10^{16} volts, with a characteristic lengthscale of 10 cm, to neutralize earth's surface gravity. While enormous, such fields are predicted by (12) to produce characteristic lengthscales in ϕ of order 10^4 km, consistent with the assumption of constant ϕ. Just as gravitational acceleration is independent of mass, so is electromagnetic antigravity. Neither couples to charge.

X. MOTION AT CONSTANT TIME

Practical interstellar travel is limited by the enormous distance scales between stars, and the finite speed of light. Therefore conceptual schemes for practical interstellar travel within the framework of general relativity rely on exotic theoretical phenomena such as worm holes, which allow "travel" from point A to point B without traversing the intervening distance. How humans could marshal the necessary gravitational fields to induce such spacetime structures is unknown. While relativistic time dilation might allow a traveller to cross the galaxy within a human lifetime, the traveler's home civilization would age much more rapidly, and the traveler would not be able to return to the home she left.

An alternate conceptual approach to this problem is to traverse the distance at constant time. This is achieved within the context of the 5D theory by using x^5 as the independent variable at constant time. The charge of particles at rest in the laboratory is interpreted within the 5D theory as motion in the fifth dimension at constant spatial position. We would like to investigate whether x^5 can serve as the independent variable for spatial derivatives in the absence of variation in time. That is, we seek geodesics in the space of x^1, x^2, x^3, x^5 that lie on slices of constant time.

Consider a charged particle at rest. Its 5D proper velocity $\widetilde{U}^a = (dx^0/d\widetilde{\tau}, dx^5/d\widetilde{\tau})$. Then $\widetilde{U}^2 = b^2$ is given by (16). Consider now motion in $(\mathbf{x}, x^5)$ for $dx^0 = 0$. Then

$$\widetilde{U}^a = \left(\frac{d\mathbf{x}}{d\widetilde{\tau}}, \frac{dx^5}{d\widetilde{\tau}}\right) = \frac{dh}{d\widetilde{\tau}}\left(\frac{d\mathbf{x}}{dh}, \alpha\right), \tag{25}$$

and

$$\widetilde{U}^2 = -\left(\frac{dh}{d\widetilde{\tau}}\right)^2 \left[\left(\frac{d\mathbf{x}}{dh}\right)^2 + \alpha^2\right] \equiv -q_5^2\left[v_5^2 + \alpha^2\right]. \tag{26}$$

Now demand that the charged particle at rest somehow changes its motion to that described by (25). Then $\widetilde{U}^2$ as described by (16) must equate to that described by (26), thereby determining the charge q_5 on geodesics of $dx^0 = 0$:

$$q_5^2 = \frac{q_0^2\alpha^2 - c^2}{v_5^2 + \alpha^2} \tag{27}$$

The cylinder condition (18) implies

$$\widetilde{U}_5 = -q_5 \left[k\mathbf{A} \cdot \mathbf{v}_5 + 1 \right] = \text{constant} \tag{28}$$

which, with (27), describes the motion through space at constant time for particles of mass m and electric charge q_0. These geodesics at constant time apparently depend on an applied vector potential $\mathbf{A}$, with associated magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$.

XI. CONCLUSIONS

Classical 5D Kaluza theory, under the simplifying assumptions of the cylinder condition and the constancy of ϕ, provides a compelling unification of general relativity and electrodynamics. By allowing for a macroscopic ¸fifth dimension, the possibility arises of motion through space at constant time, with the fifth coordinate as the independent variable. Another result of the coupling between gravity and electromagnetism is electromagnetic effects on particle motion which do not couple to electric charge, but can alter the effective gravitational force.

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